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GAS DISTRIBUTION IN A DEEP GRANULAR BED INJECTED BY FLAT JETS

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The gas-flow distribution is examined for a set of identical equidistant flat jets entering a deep immobile or fluidized bed.

There is considerable engineering interest in the distribution of the gas injected as jets into a granular bed; this applies particularly in the simulation of exchange in catalytic reactors and other equipments. Also, the gas distribution is extremely important to jet fluidization, which tends to occur with many existing gas-distributing grids, and also has a bearing on the structure of the bed near the wall in the fluidized state, as well as on the shape of any stagnant zones, and so on.

A general method has been given [1] for solving two-dimensional problems in gas distribution. Here s we use the basic assumptions of [1]: it is supposed that we can neglect the variation in the dynamic gas pressure along the jets by comparison with the pressure change within the dense phase of the bed, in which case the pressure within a jet may be taken as constant. The hydraulic resistance to the flow entering the dense phase is taken as a linear function of the infiltration speed, while the coefficient of proportionality is independent of the coordinates, i.e., we consider a linear case in infiltration theory. Since the gas speed is usually much greater than the speed of the regular particle motion in the dense phase, we consider the latter as an immobile porous body. The effects of the upper boundary of the bed are neglected, which is justified if the height of each jet is much less than the height of the bed.

The jets are considered as entering from the bottom upward and may be simulated [1] by means of a system of sections $x' = 2nLh$, $0 \le y' \le h$ ($n = 0, \pm 1, \pm 2, \ldots$) in the complex plane $z' = x' + iy'$; within the framework of this external treatment [1], the length h of a section, which characterizes the height of the jets, is taken as given a priori. Some information has been published [2] on the dependence of h on the dimensions of the injection slots, the bed parameters, and the gas speed at the slot level. Also, Lh is equal to half the distance between the jets.

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It is convenient to use the dimensionless coordinates z = x + iy, which have a scale h, in which the length of the sections is unity, while the distance between the adjacent jets is 2L. Then by analogy with [1] we obtain the following pressure distribution in the dense phase:

$$\begin{gathered}\Delta p=0,\ \partial p/\partial x=0\quad (x=0,\ y>1;\ x=L,\ y\geqslant 0),\\ p=\text{const}\,(x=0,\ 0\leqslant y\leqslant 1),\ \partial p/\partial y=-\alpha h u^{\circ}\quad (0<x<L,\ y=0),\end{gathered}\tag{1}$$

and the gas pressure in the bed unperturbed by the jets is

$$p^{\circ}=-\alpha h u^{\circ} y,\tag{2}$$

while the velocity u° of the unperturbed flow may be greater or less than the minimum fluidization velocity u_{*}.

The potential for the infiltration velocity is

$$\varphi=-\frac{p-p^{\circ}}{\alpha h},\ \mathbf{u}=\mathbf{u}^{\circ}+\mathbf{v},\ \mathbf{v}=\nabla\varphi,\tag{3}$$

and from (1) we obtain

$$\begin{gathered}\Delta\varphi=0,\ \partial\varphi/\partial x=0\quad (x=0,\ y>1;\ x=L,\ y\geqslant 0),\\ \varphi=-u^{\circ}y\,(x=0,\ 0\leqslant y\leqslant 1),\ \partial\varphi/\partial y=0\quad (0<x<L,\ y=0)\end{gathered}\tag{4}$$

(the pressure within the jet is taken as zero for reckoning the pressure).

Equations (1) and (4) are discussed within the half-strip $0 \le x \le L$, $y \ge 0$, which is quite sufficient by virtue of the obvious symmetry. This half-strip in the z = x + iy plane is mapped conformably on the upper half-plane of the $\zeta = \xi + i\eta$ complex plane by the analytical function

$$\zeta=-\cos\frac{\pi z}{L},\tag{5}$$

in which the line of section x = 0, $0 \le y \le 1$ becomes the part $-\beta \le \xi \le -1$ of the real axis in the ζ plane, where $\beta = \cosh \pi/L$.

We follow [1] and introduce the complex flow potential $\Phi = \varphi + i\psi$, where ψ is a function harmonically conjugate to φ, as well as the function $F(\zeta) = d\Phi/d\zeta$, for which we have the following Hilbert problem:

$$\begin{gathered}\operatorname{Re}F(\zeta)=f(\xi),\ -\beta\leqslant\xi\leqslant-1,\ \eta=0,\\ \operatorname{Im}F(\zeta)=0,\ \xi<-\beta,\ \xi>-1,\ \eta=0;\ \beta=\operatorname{ch}\frac{\pi}{L},\end{gathered}\tag{6}$$

where $F(\zeta)$ is an analytic function that is everywhere bounded except possibly at the points $\zeta = -1$ and $\zeta = -\beta$, where the integral is bounded [3]. In (6) we have the following function [1]:

$$f(\xi)=\frac{\partial\varphi}{\partial\xi}=\frac{\partial\varphi}{\partial y}\,\frac{\partial y}{\partial\xi}=\frac{u^{\circ}L}{\pi\sqrt{\xi^{2}-1}},\ -\beta\leqslant\xi\leqslant 1,\ \eta=0\tag{7}$$

where the corresponding boundary condition from (4) is to be used.

The Keldysh–Sedov formula [3] provides the solution to this problem; we have

$$F(\zeta)=\left[-\frac{iu^{\circ}L}{\pi^{2}}\int_{-\beta}^{-1}\frac{dt}{\sqrt{(t-1)(t+\beta)}\,(t-\zeta)}+F(\infty)\right]\sqrt{\frac{\zeta+\beta}{\zeta-1}}+C/\sqrt{(\zeta-1)(\zeta+\beta)},\tag{8}$$

where C and $F(\infty)$ are constants.

The complex velocity $U = u_x - iv_y$ is put in the form

$$U(z)=\frac{d\Phi}{dz}=\frac{d\Phi}{d\zeta}\,\frac{d\zeta}{dz}=\frac{\pi}{L}\sin\frac{\pi z}{L}\,F\left(-\cos\frac{\pi z}{L}\right).\tag{9}$$

Since U(z) is bounded at infinity, we obtain from (8) and (9) that $F(\infty) = 0$; the constant C in (8) can be derived from the obvious condition

$$\lim_{y\to\infty} v_y = -\lim_{y\to\infty} \operatorname{Im} U = \frac{Q}{2hL}, \tag{10}$$

which reflects the fact that the gas flow rate in each jet is Q. We pass to the limit $y \to \infty$ in (8) and (9) and derive the integral to obtain from (10) that

$$C = \frac{Q}{2\pi h} + \frac{2u^\circ L}{\pi^2} \operatorname{arctg}\left(\operatorname{sh}\frac{\pi}{2L}\right), \tag{11}$$

which completes the derivation of (8) and (9).

In the very simple cases $u^\circ = 0$, i.e., a jet entering an immobile granular bed, we have

$$U(z) = \frac{Q}{2hL} \sin\frac{\pi z}{L}\left[\left(1-\cos\frac{\pi z}{L}\right)\operatorname{ch}\frac{\pi}{L} - \cos\frac{\pi z}{L}\right]^{-1/2}. \tag{12}$$

We separate the real and imaginary parts in (12) to obtain expressions for the dimensionless velocity components:

$$\begin{gathered}\bar{u}_x = \frac{u_x}{u_\infty} = \frac{\sqrt{2}}{R}\left[\cos\left(\frac{\pi x}{2L}\right)\operatorname{ch}\left(\frac{\pi y}{2L}\right)\cos\lambda - \sin\left(\frac{\pi x}{2L}\right)\operatorname{sh}\left(\frac{\pi y}{2L}\right)\sin\lambda\right],\\ \bar{u}_y = \frac{u_y}{u_\infty} = \frac{\sqrt{2}}{R}\left[\left(\cos\frac{\pi x}{2L}\right)\operatorname{ch}\left(\frac{\pi y}{2L}\right)\sin\lambda + \sin\left(\frac{\pi x}{2L}\right)\operatorname{sh}\left(\frac{\pi y}{2L}\right)\cos\lambda\right],\end{gathered} \tag{13}$$

where the following quantities have been introduced:

$$R^4 = \left[\operatorname{ch}\frac{\pi}{L} - \cos\left(\frac{\pi x}{L}\right)\operatorname{ch}\left(\frac{\pi y}{L}\right)\right]^2 + \left[\sin\left(\frac{\pi x}{L}\right)\operatorname{sh}\left(\frac{\pi y}{L}\right)\right]^2,$$

$$\lambda = \frac{1}{2}\operatorname{arctg}\frac{\sin(\pi x/L)\operatorname{sh}(\pi y/L)}{\operatorname{ch}(\pi/L) - \cos\left(\frac{\pi x}{L}\right)\operatorname{ch}\left(\frac{\pi y}{L}\right)}, \quad u_\infty = \frac{Q}{2hL}.$$

If $L \to \infty$ (i.e., for a single jet), the above formulas readily give expressions for the velocity components previously derived [1].

The dimensionless quantities of (13) are shown as functions of the coordinates for L = 1 in Fig. 1a; the flow tends to become uniform as y increases, and this then has a velocity $u_x = 0$, $u_y = u_\infty$; the flow is extremely close to uniform even for y of 1.5-2, i.e., at a height above the gas-distributing system 1.5-2 times the height of each individual jet. The curves of Fig. 1a clearly characterize the establishment of the fluidized state over a slot grid if $u_y > u_*$.

In the general case, we have $u^\circ \neq 0$, and the dimensionless velocity components can be put as

$$\begin{gathered}\hat{u}_x = \hat{v}_x = u_x/u_\infty = GV_x + \bar{u}_x(1+AG),\\ \hat{u}_y = G + \hat{v}_y = u_y/u_\infty = G(1+V_y) + \bar{u}_y(1+AG),\end{gathered} \tag{15}$$

where the right sides contain the dimensionless quantities of (13) characteristic of an immobile bed without injection ($u^\circ = 0$), while the parameters are

$$G = \frac{u^\circ}{u_\infty}; \quad A = \frac{2}{\pi}\operatorname{arctg}\left(\operatorname{sh}\frac{\pi}{2L}\right) \tag{16}$$

and the vector V is defined by

$$V_x - iV_y = -\frac{i}{\pi}\sin\left(\frac{\pi z}{L}\right)\left[\frac{\beta - \cos(\pi z/L)}{1-\cos(\pi z/L)}\right]^{1/2}\int_{-\beta}^{-1}\frac{dt}{\sqrt{(t-1)(t+\beta)}\,[t+\cos(\pi z/L)]}, \quad \beta = \operatorname{ch}\frac{\pi z}{L}. \tag{17}$$

Parameter G is the ratio of the flows due to initial uniform injection with velocity u° and jet injection, respectively.

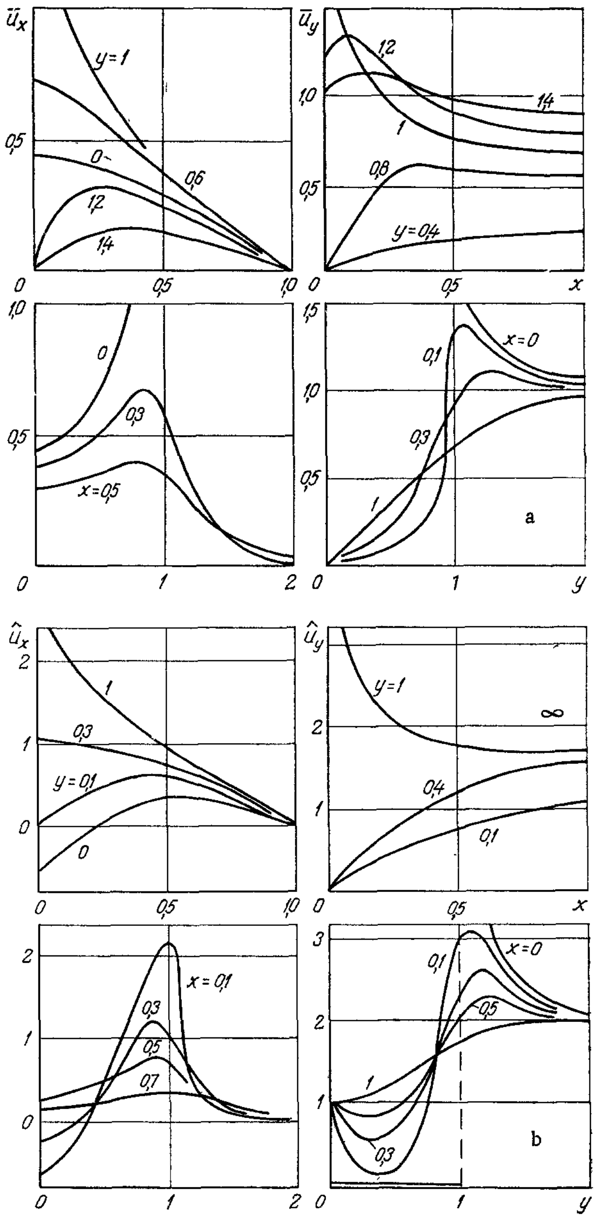

Fig. 1. Behavior of the dimensionless components of the gas velocity: a) in an immobile bed with G = 0 and L = 1; b) in an immobile or fluidized bed (G = 1 and L = 1).

In principle, the integral in (17) can be calculated by reduction to elliptic functions; however, it is more convenient for practical calculations to separate the real and imaginary parts in (17) and derive the real integrals numerically. Figure 1b shows characteristic results from (15) for x and y for L = 1 and G = 1 which were derived numerically with a BÉSM-4 computer. As in the injection of a single jet into a bed [1], we have here a form of gas injection in which the jets enter a bed unperturbed by the flow, with jet spaces in the lower part and gas escaping from the upper part. As a result, the vertical component of the speed of the overall flow is less than u_0 at heights above the grid less than h; Fig. 2 shows the critical level at which injection is replaced by ejection in relation to the parameters G and L (these results were also derived numerically).

The zones of local fluidization in an otherwise immobile bed may be examined along with possible stagnant zones in a fluidized bed in terms of the isotachs, i.e., lines in the (x, y) plane at which the vertical component of the velocity takes fixed values. These isotachs are defined in inexplicit form by

$$\hat{u}_y(x,\ y;\ G,\ L) = \Gamma, \qquad (18)$$

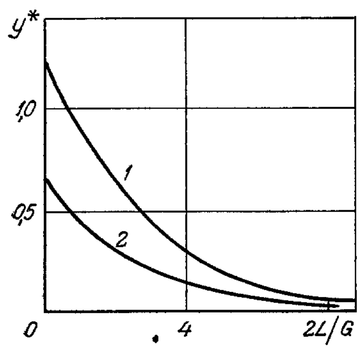

Fig. 2

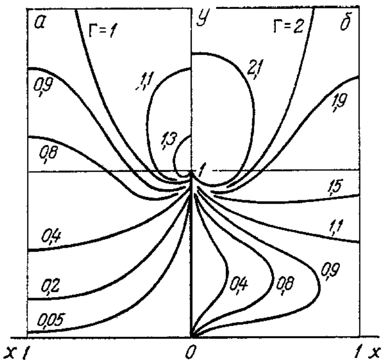

Fig. 3

Fig. 2. Critical level y* for replacement of injection by ejection in a jet as a function of 2L/G for various L: 1) $L \to \infty$; 2) L = 1.

Fig. 3. Distribution of the isotachs for the vertical component of the velocity: a) bed with G = 0; b) bed with G = 1.

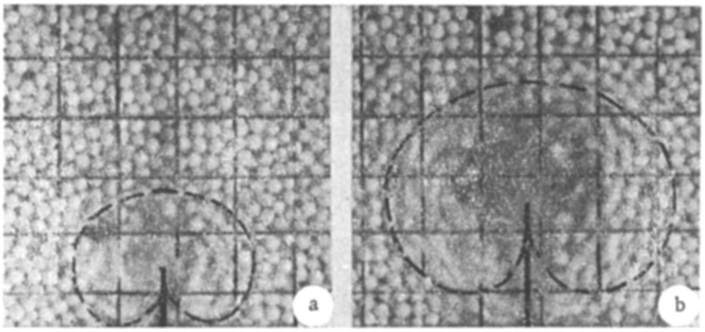

Fig. 4. Zones of particles circulation for a single gas jet entering a bed of spherical polystyrene particles of diameter 2.5 mm, slot width 1.2 mm, and Q (m^3/h) of 15 (a) 20.8 (b) and, bed height, 150 mm; grid spacing, 1 cm. The dashed line is the theoretical boundary of the circulation zone constructed in a section representing the effective height of the jet.

where $\hat{u}_y$ is defined in (15) and Γ is a parameter; the pattern of the isotachs for an immobile bed (G = 0) is essentially different from that of an infiltrated or fluidized bed (G > 0).

In the first case (Fig. 3a), with Γ > 1, the isotachs are closed curves tangential at the upper points to lines that represent the jets, and they resemble the lines of constant velocity discussed in [1]. The isotach corresponding to Γ = 1 recedes to infinity and asymptotically approaches the straight lines x = ±L (for a jet along the line x = 0). The isotach for Γ < 1 intersects these straight lines at a finite value of the dimensionless coordinate y, where it meets the isotachs arising from adjacent jets. Clearly, the isotachs lie along the base y = 0 of the bed for Γ = 0. In $u_* = \Gamma_* u_\infty$, the region within a closed isotach or that above the unclosed isotach corresponding to the critical value Γ_* may represent local fluidization and ascending motion of the granular material.

In the second case, we have G > 0 and, therefore, $u^\circ \neq 0$; closed isotachs arise for Γ sufficiently small, which lie around the jets. If Γ_* corresponds to such an isotach, the granular material will be fluidized everywhere outside the stagnant zones, i.e., areas within the isotachs that directly adjoin the jets. The material is such a stagnant zone is either immobile or else slides downward as a body. Figure 3b shows the isotach pattern for G = 1.

If $u^\circ + u_\infty$ is less than u_*, a closed local-fluidization zone or circulation area is formed around each jet; these adjacent zones clearly come together as $u^\circ + u_\infty$ increases. It is then possible, in principle, for the immobile parts between adjacent zones to be broken up on account of various random factors, i.e., one obtains a partially fluidized bed, with a granular material fluidized in a certain region directly above the jets, whose thickness is roughly equal to the vertical scale of the local fluidization zones, while there is no fluidization above and below that area.

We collaborated with G. A. Minaev and S. M. Ellengorn in performing a series of measurements on the cavities and circulation zones formed near single and multiple jets in order to derive a fuller physical picture of the processes occurring in a granular bed on jet injection; transparent models for immobile and fluidized beds were used. The curves of Fig. 3 represent the shapes of the zones closely. Details of the results a these experiments will be presented elsewhere, but Fig. 4 shows results for a single jet entering an immobile bed, along with the theoretical boundaries to the circulation zone.

NOTATION

A	is the parameter in (16);
C	is the constant defined in (11);
F	is the analytical function from solution of Hilbert equation in (6);
$f(\xi)$	is the function defined in (7);
h	is the effective jet height;
L	is the half dimensionless distance between jets;
p	is the gas pressure;
Q	is the gas flow rate in jet;
$\dot{R}$	is the parameter in (14);
U	is the complex velocity;
u	is the total gas flow velocity;
u_∞	is the average velocity of flow due to jets;
V	is the vector defined by (17);
v	is the flow velocity due to jets;
x', y'	are the coordinates;
$z = x + iy$	is the complex plane of dimensionless coordinates;
α	is the hydraulic resistance coefficient of dense phase;
β	is the parameter defined in (6);
Γ	is the parameter in (18);
$\zeta = \xi + i\eta$	is the complex plane of coordinates defined by function (5);
λ	is the angle defined in (14);
Φ	is the filtration flow potential;
ψ	is the function harmonically conjugate to φ.

Indices

$^\circ$	is the flow unperturbed by jets;
$*$	is the minimal fluidization;
$-$, $\wedge$	are the dimensionless velocities.

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